VISCOSITY

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Fluid - liquid, gas or powder that is flowable, that can changes its shape and movement.
During the movement of molecules of gas-liquid relative to one another - the flow resistance of the braking movement is referred to as internal friction and is defined as the viscosity.

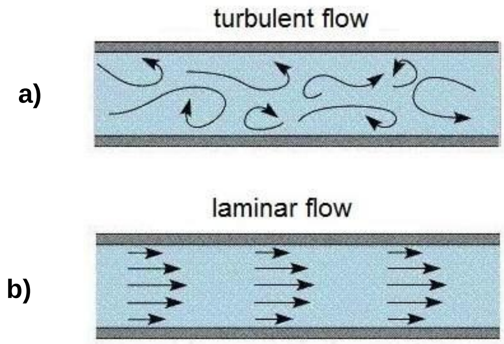

Fig.1. The flow of liquid through the tube: a) turbulent flow and b) laminar flow.

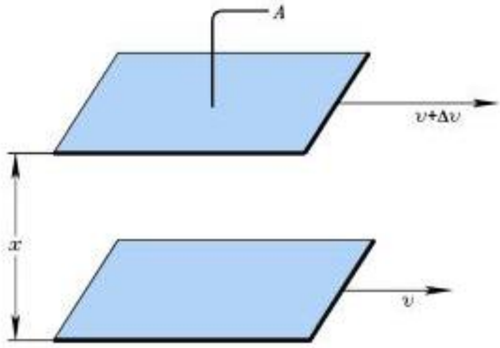

Fig.2. the liquid layers moving relative to one another

In order to maintain a constant speed difference dV is necessary to overcome the forces of intermolecular attraction tangential force F given by the equation:

$$F = \eta \cdot A \frac{dV}{dr} \quad (1)$$

where: F - the force acting between two parallel layers of area A, one of which moves faster by Δv remaining at the distance x for a liquid with a viscosity **η** (read: eta).

Solving this equation for viscosity we can receive unit of dynamic viscosity [P] – poise, which is equal to the force required to give a difference in speed of two layers of liquids with surface $1 cm^2$ spaced apart by 1 cm amount to $1 cm/s^2$. The unit of viscosity in the SI units system is $N \cdot s/m^2$ = kg /m·s. The inverse of the dynamic viscosity is the fluidity **φ** (read fi):

$$\varphi = \frac{1}{\eta} \quad (2)$$

The kinematic viscosity **v** is defined as the ratio of the dynamic viscosity to density of liquid:

$$v = \frac{\eta}{d} \qquad [P/g \cdot cm^{-3} = 1 St] \quad (3)$$

where: P – poise, St – stokes.

Poiseuille analyzed the flow, referring to a circular tube and formulated the dependence of the force F of flow parameters. In this case, the force F is given by the equation:

$$F = \eta \cdot 2\pi \cdot r \frac{dV}{dr} \quad (4)$$

where: r – radius of the tube, l – length of the tube, V – velocity of the flow.

The velocity of the liquid stream in the tube section is not the same in whole volume of the fluid, the flow rate is the largest at the center of the tube and decreases to zero at the wall. It can therefore be seen spatially fluid movement as a movement of concentric tubes of radius r <R, which is shown in Figure 2.

The flow velocity **V** at distance **r** from the tube wall for a pipe having a radius **R** at a pressure **p** across a distance **l** and viscosity **η** is equal:

$$V = \frac{p}{4\eta l}(R^2 - r^2) \quad (5)$$

The above equation can be used with certain restrictions. Namely, describes correctly the flow through a small diameter pipe and a small flow rate when the velocity distribution is consistent with the distribution shown in Figure 1b. This flow is called laminar or viscous flow. The turbulent flow shown schematically in the form of vortices in Fig. 1b. occurs in pipes with large diameter and large speed of the fluid.

The nature of the flow is usually determined empirically using dimensionless value - **Reynolds number** defined by the formula:

$$Re = \frac{D \cdot V \cdot d}{\eta} \tag{6}$$

where: D - diameter of the tube, V - average speed of the fluid in the pipe, d - the liquid density, η - viscosity of the fluid.

As stated experimentally when the Reynolds number is smaller than 2300, the flow is laminar, whereas for values larger than 4000 becomes turbulent. Characterization the of flow of Reynolds number between these values is difficult to determine.

Equation (4) refers to the so-called Newtonian fluids, that is, the viscosity of which does not depend on the velocity gradient.

Measurement of viscosity can be done by several methods classified in the two groups. One of these methods are based on the Poiseuille law, and measure the rate of flow through the capillary tube. The second group are methods based on Stokes equation - measuring the rate of descent balls in the liquid.

Capillary viscometers operating principle is based on the Poiseuille law, according to which the liquid volume **V** passing at time **t** by a capillary of radius **r** and length **l** under the pressure difference **Δp** is equal:

$$V = \frac{\pi \cdot r^4 \cdot \Delta p \cdot t}{8\eta \cdot l} \tag{7}$$

An example of a capillary viscometer is the Ostwald viscometer (Figure 2), wherein the liquid flows through the capillary tube under the influence of hydrostatic pressure difference between the two arms of U-tube, namely:

$$\Delta p = (h_1 - h_2)dg \tag{8}$$

wherein: (h_1-h_2), - the difference in liquid levels in the two arms of U-tube, d - the liquid density, g - acceleration due to gravity.

Direct use of above equation (7) is inconvenient due to the necessity of determining many parameters affected by measurement error. In practice, for the determination of the viscosity, there is measured time of the liquid flow through a capillary between the levels **a** and **b** (Fig.3). If time of flow for the standard liquid (eg. water) of known viscosity is $\mathbf{t_0}$ and for tested liquid $\mathbf{t_x}$ and we know that $\mathbf{V_0 = V_x}$, we can calculate the viscosity of tested fluid from the equation (9):

$$\eta_x = \eta_0 \cdot \frac{d_x \cdot t_x}{d_0 \cdot t_0} \tag{9}$$

To keep level difference (a - b) the same, the Ostwald viscometer must be always filled with the same volume of liquid. This disadvantage is eliminated in a modified Ostwald viscometer - Ubbelohde viscometer, which we use in exercise.

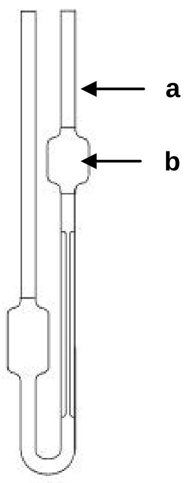

Fig.3. Ostwald capillary viscometer.

Fig.4. Ubbelohde capillary viscometer.

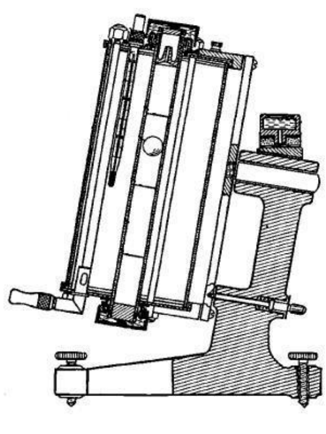

Fig.5. Höppler viscometer.

Method of measuring the rate of descent ball in liquid was developed by Stokes which brought the formula for the force with which the viscous medium of a density **d** resists movement of the ball of radius **r** density $\mathbf{d_k}$ and velocity **V**. This force is equal $6\pi\eta rV$ and when it balances the apparent weight of ball (gravity less buoyancy):

$$\frac{4}{3}\pi r^3 \cdot (d_k\text{-}d)\cdot g \qquad (10)$$

The ball falls the uniform motion at a constant speed of V_0. Comparison of the two forces:

$$6\pi\eta r V_0 = \frac{4}{3}\cdot\pi r^3 \cdot (d_k\text{-}d)\cdot g \qquad (11)$$

It leads to the formula for the viscosity of the liquid:

$$\eta = \frac{2}{9}\cdot\frac{r^2}{V_0}\cdot(d_k\text{-}d)\cdot g \qquad (12)$$

The viscosity measurement method based on Stokes law is a Höppler viscometer (Fig. 5). The viscometer measures the time at which the ball is pass the same way **(a)** between the selected lines in standard liquid **(t_0)** and in tested liquid **(t_x).** Depending on the density and viscosity, there are selected glass or metal balls having the volume and density such that the rate of descent could be easily measured. Knowing that:

$$V_0 = \frac{a}{t} \qquad (13)$$

and taking into account equation (12) we receive the formula:

$$\eta_x = \eta_0 \cdot[(d_k\text{-}d_x)\cdot t_x / (d_k\text{-}d_0)\cdot t_0] \qquad (14)$$

Table 1. Viscosity of selected liquids (at 25 °C unless otherwise specified)

Liquid:	**Viscosity [cP = P · 10^{-2}]**
liquid nitrogen (77K)	0.158
acetone	0.306
methanol	0.544
water	0.894
ethanol	1.074
mercury	1.526
nitrobenzene	1.863
ethylene glycol	16.1
motor oil SAE 10 (20 °C)	65
motor oil SAE 40 (20 °C)	319
glycerol (at 20 °C)	1 200
blood (37 °C)	3–4
honey	2 000–10 000
molten chocolate	45 000–130 000
molten glass	10 000–1 000 000